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A CHARACTERIZATION OF QUASI-IDEALS IN Γ-SEMIGROUPS

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Abstract

It is well known that any semigroup can be reduced to a Γ-semigroup. The aim of this paper is to study the concept of (0-) minimal and maximal quasi-ideals in Γ-semigroups, and give some characterizations of (0-) minimal and maximal quasi-ideals in Γ-semigroups analogous to the characterizations of minimal and maximal left ideals in ordered semigroups considered by Cao and Xu [2].

1. Introduction and Prerequisites

Let S be a semigroup. A subsemigroup Q of S is called a *quasi-ideal* of S if $SQ \cap QS \subseteq Q$. The definitions of a minimal quasi-ideal and a 0-minimal quasi-ideal in semigroups in [12] are given differently as follows: A quasi-ideal Q of a semigroup S without zero is called a *minimal quasi-ideal* of S if Q does not properly contain any quasi-ideal of S. For a

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semigroup S with zero, a 0-*minimal quasi-ideal* of S is a nonzero quasi-ideal of S which does not properly contain any nonzero quasi-ideal of S. The notion of a quasi-ideal in semigroups was first introduced by Steinfeld [10] in 1956 and it has been widely studied. In 1956, Steinfeld [11] gave some characterizations of 0-minimal quasi-ideals in semigroups. In 1981, the concept and notion of the $\Gamma-$semigroup was introduced by Sen [8]. In 2000, Cao and Xu [2] characterized the minimal and maximal left ideals in ordered semigroups, and gave some characterizations of minimal and maximal left ideals in ordered semigroups. In 2002, Arslanov and Kehayopulu [1] gave some characterizations of minimal and maximal ideals in ordered semigroups. In 2004, Iampan and Siripitukdet [5] characterized the (0-)minimal and maximal ordered left ideals in po-Γ-semigroups, and gave some characterizations of (0-)minimal and maximal ordered left ideals in po-Γ-semigroups. In 2006, the concept and notion of a quasi-Γ-ideal in Γ-semigroups was introduced by Chinram [3].

The concept of a (0-)minimal and maximal one-sided ideal or ideal is the really interested and important thing in (ordered) semigroups and (po-)Γ-semigroups. We can see that the notion of a one-side ideal is a generalization of the notion of an ideal, and the notion of a quasi-ideal is a generalization of the notion of a one-side ideal. Hence we also characterize the (0-)minimal and maximal quasi-ideals in Γ-semigroups, and give some characterizations of (0-)minimal and maximal quasi-ideals in Γ-semigroups.

To present the main results we first recall some definitions which are important here.

Let M and Γ be any two nonempty sets. M is called a Γ-*semigroup* [8] if there exists a mappings $M \times \Gamma \times M \to M,$ written as $(a, \gamma, b) \mapsto a\gamma b,$ satisfying the following identity $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma.$ A nonempty subset K of M is called a *sub*-Γ-*semigroup* of M if $a\gamma b \in K$ for all $a, b \in K$ and $\gamma \in \Gamma.$ For nonempty subsets A, B of

M, let $A\Gamma B := \{a\gamma b \mid a \in A,\ b \in B \text{ and } \gamma \in \Gamma\}$. We also write $a\Gamma B$, $A\Gamma b$ and $a\Gamma b$ for $\{a\}\Gamma B$, $A\Gamma\{b\}$ and $\{a\}\Gamma\{b\}$, respectively.

Examples of Γ-semigroups can be seen in [5, 7] and [9] respectively.

The following example comes from Dixit and Dewan [4].

Example 1.1. Let $T = \{-i, 0, i\}$ and $\Gamma = T$. Then T is a Γ-semigroup under the multiplication over complex number while T is not a semigroup under complex number multiplication.

A nonempty subset I of a Γ-semigroup M is called an *ideal* of M if $M\Gamma I \subseteq I$ and $I\Gamma M \subseteq I$. A sub-Γ-semigroup Q of a Γ-semigroup M is called a *quasi-ideal* of M if $M\Gamma Q \cap Q\Gamma M \subseteq Q$. A sub-$\Gamma$-semigroup B of a Γ-semigroup M is called a *bi-ideal* of M if $B\Gamma M\Gamma B \subseteq B$. Then the notion of a quasi-ideal is a generalization of the notion of an ideal, and the notion of a bi-ideal is a generalization of the notion of a quasi-ideal. The intersection of all quasi-ideals of a sub-Γ-semigroup K of a Γ-semigroup M containing a nonempty subset A of K is called the *quasi-ideal of K generated by A*. For $A = \{a\}$, let $Q_K(a)$ denote the quasi-ideal of K generated by $\{a\}$. If $K = M$, then we also write $Q_M(a)$ as $Q(a)$. An element a of a Γ-semigroup M with at least two elements is called a *zero element* of M if $x\gamma a = a\gamma x = a$ for all $x \in M$ and $\gamma \in \Gamma$, and denote it by 0. If M is a Γ-semigroup with zero, then every quasi-ideal of M contains a zero element. A Γ-semigroup M without zero is called *Q-simple* if it has no proper quasi-ideals. A Γ-semigroup M with zero is called *0-Q-simple* if it has no nonzero proper quasi-ideals and $M\Gamma M \neq \{0\}$. A quasi-ideal Q of a Γ-semigroup M without zero is called a *minimal quasi-ideal* of M if there is no quasi-ideal A of M such that $A \subset Q$. Equivalently, if for any quasi-ideal A of M such that $A \subseteq Q$, we have $A = Q$. A nonzero quasi-ideal Q of a Γ-semigroup M with zero is called a 0-*minimal quasi-ideal* of M if there is no nonzero quasi-ideal A of M such that $A \subset Q$. Equivalently, if for any nonzero quasi-ideal A of M such that $A \subseteq Q$, we have $A = Q$. Equivalently, if for any quasi-ideal A of M such that $A \subset Q$, we have $A = \{0\}$. A proper quasi-ideal Q of a Γ- semigroup M is called a *maximal quasi-ideal* of M if for any quasi-ideal A of M such that $Q \subset A$, we have $A = M$. Equivalently, if for any proper quasi-ideal A of M such that $Q \subseteq A$, we have $A = Q$.

Our aim in this paper is fourfold.

1. To introduce the concept of a Q-simple and 0-Q-simple Γ- semigroup.

2. To characterize the properties of quasi-ideals in Γ- semigroups.

3. To characterize the relationship between (0-)minimal quasi-ideals and (0-)Q-simple Γ- semigroups.

4. To characterize the relationship between maximal quasi-ideals and (0-)Q-simple Γ- semigroups.

We shall assume throughout this paper that M stands for a Γ- semigroup. Before the characterizations of quasi-ideals for the main theorems, we give some auxiliary results which are necessary in what follows. We begin by recalling the following lemma comes from Chinram [3].

Lemma 1.2. *For any* $a \in M$,

$$Q(a) = (M\Gamma a \cap a\Gamma M) \cup \{a\}.$$

The following two lemmas are also necessary for our considerations, and easy to verify.

Lemma 1.3. *For any* $a \in M$, $M\Gamma a \cap a\Gamma M$ *is a quasi-ideal of M.*

Lemma 1.4. *Let* $\{Q_\gamma \mid \gamma \in \Lambda\}$ *be a collection of quasi-ideals of M. Then* $\bigcap_{\gamma \in \Lambda} Q_\gamma$ *is a quasi-ideal of M if* $\bigcap_{\gamma \in \Lambda} Q_\gamma \neq \varnothing$.

Lemma 1.5. *If M has no zero element, then the following statements are equivalent:*

(i) *M is Q-simple.*

(ii) $M\Gamma a \cap a\Gamma M = M$ *for all* $a \in M$.

(iii) $Q(a) = M$ *for all* $a \in M$.

Proof. Since M is Q-simple, it follows from Lemma 1.3 that $M\Gamma a \cap a\Gamma M = M$ for all $a \in M$. Therefore (i) implies (ii). By Lemma 1.2, we have $Q(a) = (M\Gamma a \cap a\Gamma M) \cup \{a\} = M \cup \{a\} = M$ for all $a \in M$. Thus (ii) implies (iii). Now, let Q be a quasi-ideal of M, and let $a \in Q$. Then $M = Q(a) \subseteq Q \subseteq M$, so $Q = M$. Hence M is Q-simple, we have that (iii) implies (i).

Lemma 1.6. *If M has a zero element, then the following statements hold:*

(i) *If M is* 0-*Q-simple, then* $Q(a) = M$ *for all* $a \in M \setminus \{0\}$.

(ii) *If* $Q(a) = M$ *for all* $a \in M \setminus \{0\}$, *then either* $M\Gamma M = \{0\}$ *or M is* 0-*Q-simple.*

Proof. (i) Assume that M is 0-Q-simple. Then, since $Q(a)$ is a nonzero quasi-ideal of M for all $a \in M \setminus \{0\}$, $Q(a) = M$ for all $a \in M \setminus \{0\}$.

(ii) Assume that $Q(a) = M$ for all $a \in M \setminus \{0\}$ and $M\Gamma M \neq \{0\}$. Now, let Q be a nonzero quasi-ideal of M, and let $a \in Q \setminus \{0\}$. Then $M = Q(a) \subseteq Q \subseteq M$, so $Q = M$. Therefore M is 0-Q-simple.

Lemma 1.7. *If Q is a quasi-ideal of M, and K is a sub-*Γ*-semigroup of M, then the following statements hold:*

(i) *If K is Q-simple such that* $K \cap Q \neq \varnothing$, *then* $K \subseteq Q$.

(ii) *If K is* 0-*Q-simple such that* $K \setminus \{0\} \cap Q \neq \varnothing$, *then* $K \subseteq Q$.

Proof. (i) Assume that K is Q-simple such that $K \cap Q \neq \varnothing$, and let $a \in K \cap Q$. Then, by Lemma 1.3, $K\Gamma a \cap a\Gamma K$ is a quasi-ideal of K.

Hence $K\Gamma a \cap a\Gamma K = K$. Therefore $K = K\Gamma a \cap a\Gamma K \subseteq M\Gamma Q \cap Q\Gamma M \subseteq Q$, so $K \subseteq Q$.

(ii) Assume that K is 0-Q-simple such that $K \setminus \{0\} \cap Q \neq \varnothing$, and let $a \in K \setminus \{0\} \cap Q$. Then, by Lemmas 1.2 and 1.6 (i), $K = Q_K(a) = (K\Gamma a \cap a\Gamma K) \cup \{a\} \subseteq (M\Gamma a \cap a\Gamma M) \cup \{a\} = Q(a) \subseteq Q$. Hence $K \subseteq Q$.

Hence the proof is completed.

We now give the main theorem of this paper as bellow.

2. (0-) Minimal Quasi-ideals

The aim of this section is to characterize the relationship between minimal quasi-ideals and Q-simple Γ-semigroups, and 0-minimal quasi-ideals and 0-Q-simple Γ-semigroups.

Theorem 2.1. *If M has no zero element, and Q is a quasi-ideal of M, then the following statements hold:*

(i) *If Q is an ideal of M and a minimal quasi-ideal without zero of M, then either there exists a quasi-ideal A of Q such that $Q\Gamma A \cap A\Gamma Q = \varnothing$ or Q is Q-simple.*

(ii) *If Q is Q-simple, then Q is a minimal quasi-ideal of M.*

(iii) *If Q is an ideal of M and a minimal quasi-ideal with zero of M, then either there exists a nonzero quasi-ideal A of Q such that $Q\Gamma A \cap A\Gamma Q = \{0\}$ or Q is 0-Q-simple.*

Proof. (i) Assume that an ideal Q is a minimal quasi-ideal without zero of M, and let $Q\Gamma A \cap A\Gamma Q \neq \varnothing$ for all quasi-ideals A of Q. Clearly, Q is a *sub*-Γ-semigroup of M. Now, let A be a quasi-ideal of Q. Then $\varnothing \neq Q\Gamma A \cap A\Gamma Q \subseteq A$. Define $H := \{h \in A \mid h \in Q\Gamma A \cap A\Gamma Q\}$. Then $\varnothing \neq H \subseteq A \subseteq Q$. To show that H is a quasi-ideal of M, let $h_1, h_2 \in H$ and $\gamma \in \Gamma$. Then $h_1 = q_1\beta_1 a_1 = a_1'\beta_1' q_1'$ and $h_2 = q_2\beta_2 a_2 = a_2'\beta_2' q_2'$ for some $a_1, a_1', a_2, a_2' \in A$, $q_1, q_1', q_2, q_2' \in Q$ and $\beta_1, \beta_1', \beta_2, \beta_2' \in \Gamma$, so

$h_1\gamma h_2 = q_1\beta_1a_1\gamma q_2\beta_2a_2 = a_1'\beta_1'q_1'\gamma a_2'\beta_2'q_2'$. Since A is a quasi- ideal of Q, A is a bi-ideal of Q. Thus $A\Gamma Q\Gamma A \subseteq A$, so $a_1\gamma q_2\beta_2a_2$, $a_1'\beta_1'q_1'\gamma a_2' \in A$. Since $h_1\gamma h_2 \in H\Gamma H \subseteq A\Gamma A \subseteq A$, we have $h_1\gamma h_2 \in H$. Hence H is a *sub*-Γ- semigroup of M. If $x \in M\Gamma H \cap H\Gamma M$, then $x = m\gamma h = h'\gamma' m'$ for some $m, m' \in M$, $h, h' \in H$ and $\gamma, \gamma' \in \Gamma$. Thus $h = q_1\beta_1a_1 = a_2\beta_2q_2$ and $h' = q_1'\beta_1'a_1' = a_2'\beta_2'q_2'$ for some $a_1, a_1', a_2, a_2' \in A$, $q_1, q_1', q_2, q_2' \in Q$ and $\beta_1, \beta_1', \beta_2, \beta_2' \in \Gamma$. Hence $x = m\gamma h = m\gamma q_1\beta_1a_1$ and $x = h'\gamma'm' = a_2'\beta_2'q_2'\gamma'm'$. Since Q is an ideal of M, we have $m\gamma q_1, q_2'\gamma'm' \in Q$. Thus $x \in Q\Gamma A \cap A\Gamma Q \subseteq A$. Hence $x \in H$, so $M\Gamma H \cap H\Gamma M \subseteq H$. Therefore H is a quasi-ideal of M. Since Q is a minimal quasi-ideal of M, we get $H = Q$. Therefore $A = Q$, so Q is Q-simple.

(ii) Assume that Q is Q-simple, and let A be a quasi-ideal of M such that $A \subseteq Q$. Then $A \cap Q \neq \varnothing$, it follows from Lemma 1.7 (i) that $Q \subseteq A$. Hence $A = Q$, so Q is a minimal quasi-ideal of M.

(iii) Similar to the proof of statement (i).

Therefore we complete the proof of the theorem.

Using the same proof of Theorem 2.1 (i) and Lemma 1.7 (ii), we have Theorem 2.2.

Theorem 2.2. *If M has a zero element, and Q is a nonzero quasi-ideal of M, then the following statements hold:*

(i) *If Q is an ideal of M and a* 0-*minimal quasi-ideal of M, then either there exists a nonzero quasi-ideal A of Q such that* $Q\Gamma A \cap A\Gamma Q = \{0\}$ *or Q is* 0-*Q-simple.*

(ii) *If Q is* 0-*Q-simple, then Q is a* 0-*minimal quasi-ideal of M.*

Theorem 2.3. *If M has no zero element but it has a proper quasi-ideal, then every proper quasi-ideal of M is minimal if and only if the intersection of any two distinct proper quasi- ideals is empty.*

Proof. Assume Q_1 and Q_2 are two distinct proper quasi-ideals of M. Then Q_1 and Q_2 are minimal. If $Q_1 \cap Q_2 \neq \varnothing$, then $Q_1 \cap Q_2$ is a quasi-ideal of M by Lemma 1.4. Since Q_1 and Q_2 are minimal, $Q_1 = Q_2$, It is a contradiction. Hence $Q_1 \cap Q_2 = \varnothing$.

The converse is obvious.

Using the same proof of Theorem 2.3, we have Theorem 2.4.

Theorem 2.4. *If M has a zero element and a nonzero proper quasi-ideal, then every nonzero proper quasi-ideal of M is* 0-*minimal if and only if the intersection of any two distinct nonzero proper quasi-ideals is* $\{0\}$.

3. Maximal Quasi-ideals

The aim of this section is to characterize the relationship between maximal quasi-ideals and the set $\mathcal{U}$ in Γ- semigroups.

Theorem 3.1. *Let Q be a quasi-ideal of M. If either $M \setminus Q = \{a\}$ for some $a \in M$ or $M \setminus Q \subseteq M\Gamma b \cap b\Gamma M$ for all $b \in M \setminus Q$, then Q is a maximal quasi-ideal of M.*

Proof. Let A be a quasi-ideal of M such that $Q \subset A$. Then we consider the following two cases:

Case 1: $M \setminus Q = \{a\}$ for some $a \in M$.

Then $M = Q \cup \{a\}$. Since $Q \subset A$, $\varnothing \neq A \setminus Q \subseteq M \setminus Q = \{a\}$. Hence $A \setminus Q = \{a\}$, so $A = Q \cup \{a\} = M$.

Case 2: $M \setminus Q \subseteq M\Gamma b \cap b\Gamma M$ for all $b \in M \setminus Q$.

If $b \in A \setminus Q \subseteq M \setminus Q$, then $M \setminus Q \subseteq M\Gamma b \cap b\Gamma M \subseteq M\Gamma A \cap A\Gamma M \subseteq A$. Hence $M = Q \cup M \setminus Q \subseteq Q \cup A = A$, so $A = M$

Therefore Q is a maximal quasi-ideal of M.

Theorem 3.2. *If Q is a maximal quasi-ideal of M, and $Q \cup Q(a)$ is a quasi-ideal of M for all $a \in M \setminus Q$, then either*

(i) $M \setminus Q = \{a\}$ *and* $M\Gamma a \cap a\Gamma M \subseteq Q$ *for some* $a \in M \setminus Q$ *or*

(ii) $M \setminus Q \subseteq Q(a)$ *for all* $a \in M \setminus Q$.

Proof. Assume that Q is a maximal quasi-ideal of M, and $Q \cup Q(a)$ is a quasi-ideal of M for all $a \in M \setminus Q$. Then we have the following two cases:

Case 1: $M\Gamma a \cap a\Gamma M \subseteq Q$ for some $a \in M \setminus Q$.

Since $Q \cup \{a\} = Q \cup (M\Gamma a \cap a\Gamma M) \cup \{a\} = Q \cup Q(a)$, we have $Q \cup \{a\}$ is a quasi-ideal of M. Since $a \in M \setminus Q$, we have $Q \subset Q \cup \{a\}$. Thus $Q \cup \{a\} = M$ because Q is a maximal quasi-ideal of M, so $M \setminus Q \subseteq \{a\}$. Hence $M \setminus Q = \{a\}$. In this case, the condition (i) is satisfied.

Case 2: $M\Gamma a \cap a\Gamma M \not\subseteq Q$ for all $a \in M \setminus Q$.

If $a \in M \setminus Q$, then $Q \subset Q \cup (M\Gamma a \cap a\Gamma M) \subseteq Q \cup Q(a)$ by Lemma 1.2. Since $Q \cup Q(a)$ is a quasi-ideal of M, and Q is a maximal quasi-ideal of M, we get $Q \cup Q(a) = M$. Therefore $M \setminus Q \subseteq Q(a)$ for all $a \in M \setminus Q$. In this case, the condition (ii) is satisfied.

Hence the proof is completed.

For a Γ-semigroup M, let $\mathcal{U}$ denote the union of all nonzero proper quasi-ideals of M if M has a zero element, and let $\mathcal{U}$ denote the union of all proper quasi-ideals of M if M has no zero element. Then it is easy to verify Lemma 3.3.

Lemma 3.3. $M = \mathcal{U}$ *if and only if* $Q(a) \neq M$ *for all* $a \in M$.

As a consequence of Theorem 3.2 and Lemma 3.3, we obtain Theorem 3.4.

Theorem 3.4. *If M has no zero element, then one of the following four conditions is satisfied:*

(i) $\mathcal{U}$ *is not a quasi-ideal of M.*

(ii) $Q(a) \neq M$ *for all* $a \in M$.

(iii) *There exists* $a \in M$ *such that* $Q(a) = M$, $a \notin M\Gamma a \cap a\Gamma M$ *and*

$M \setminus \mathcal{U} = \{a\}$, M is not Q-simple, and $\mathcal{U}$ is the unique maximal quasi-ideal of M.

(iv) *$M \setminus \mathcal{U} \subseteq Q(a)$ for all $a \in M \setminus \mathcal{U}$, M is not Q-simple, $M \setminus \mathcal{U} = \{x \in M \mid Q(x) = M\}$, and $\mathcal{U}$ is the unique maximal quasi-ideal of M.*

Proof. Assume that $\mathcal{U}$ is a quasi-ideal of *M*. Then $\mathcal{U} \neq \varnothing$. Thus we consider the following two cases:

Case 1: $\mathcal{U} = M$.

By Lemma 3.3, we have $Q(a) \neq M$ for all $a \in M$. In this case, the condition (ii) is satisfied.

Case 2: $\mathcal{U} \neq M$.

Then M is not Q-simple. To show that $\mathcal{U}$ is the unique maximal quasi-ideal of *M*, let *A* is a quasi-ideal of *M* such that $\mathcal{U} \subset A$. If $A \neq M$, then *A* is a proper quasi-ideal of *M*. Thus $A \subseteq \mathcal{U}$, so it is a contradiction. Hence $\mathcal{U}$ is a maximal quasi-ideal of *M*. Next, assume that Q is a maximal quasi-ideal of *M*. Then $Q \subseteq \mathcal{U} \subset M$ because Q is a proper quasi-ideal of *M*. Since Q is a maximal quasi-ideal of *M*, we have $Q = \mathcal{U}$. Hence $\mathcal{U}$ is the unique maximal quasi-ideal of *M*. Since $\mathcal{U} \neq M$, it follows from Lemma 3.3 that $Q(x) = M$ for some $x \in M$. Clearly, $Q(x) = M$ for all $x \in M \setminus \mathcal{U}$. Thus $M \setminus \mathcal{U} = \{x \in M \mid Q(x) = M\}$, so $\mathcal{U} \cup Q(x) = M$ is a quasi-ideal of *M* for all $x \in M \setminus \mathcal{U}$. By Theorem 3.2, we have the following two cases:

(i) $M \setminus \mathcal{U} = \{a\}$ and $M\Gamma a \cap a\Gamma M \subseteq \mathcal{U}$ for some $a \in M \setminus \mathcal{U}$ or

(ii) $M \setminus \mathcal{U} \subseteq Q(a)$ for all $a \in M \setminus \mathcal{U}$.

Assume $M \setminus \mathcal{U} = \{a\}$ and $M\Gamma a \cap a\Gamma M \subseteq \mathcal{U}$ for some $a \in M \setminus \mathcal{U}$. If $a \in M\Gamma a \cap a\Gamma M$, then $a \in \mathcal{U}$. It is a contradiction. Hence $a \notin M\Gamma a \cap a\Gamma M$. In this case, the condition (iii) is satisfied. Now, assume $M \setminus \mathcal{U} \subseteq Q(a)$ for all $a \in M \setminus \mathcal{U}$. In this case, the condition (iv) is satisfied.

Hence the theorem is now completed.

Using the same proof of Theorem 3.5, we have Theorem 3.5.

Theorem 3.5. *If M has a zero element and* $M\Gamma M \neq \{0\}$, *then one of the following five conditions is satisfied:*

(i) $\mathcal{U}$ *is not a quasi-ideal of M.*

(ii) $Q(a) \neq M$ *for all* $a \in M$.

(iii) $\mathcal{U} = \{0\}$, $M \setminus \mathcal{U} = \{x \in M \mid Q(x) = M\}$, *and* $\mathcal{U}$ *is the unique maximal quasi-ideal of M.*

(iv) *There exists* $a \in M$ *such that* $Q(a) = M$, $a \notin M\Gamma a \cap a\Gamma M$ *and* $M \setminus \mathcal{U} = \{a\}$, M *is not* 0-*Q-simple, and* $\mathcal{U}$ *is the unique maximal quasi-ideal of M.*

(v) $M \setminus \mathcal{U} \subseteq Q(a)$ *for all* $a \in M \setminus \mathcal{U}$, M *is not* 0-*Q-simple,* $M \setminus \mathcal{U} = \{x \in M \mid Q(x) = M\}$, *and* $\mathcal{U}$ *is the unique maximal quasi-ideal of M.*

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